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Phil. Trans. R. Soc. Lond. A 1956 **248**, 609-620
doi: 10.1098/rsta.1956.0011

THE DISTRIBUTION OF ELECTRONS IN THE UNDISTURBED F_2 LAYER OF THE IONOSPHERE

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(*Communicated by J. A. Ratcliffe, F.R.S.—Received* 17 *June* 1955)

CONTENTS

$h'(f)$ curves recorded at Watheroo, Huancayo and Slough near midwinter, midsummer and equinox in years of sunspot maximum and minimum were analyzed so as to provide $N(h)$ curves which show the electron density (N) as a function of the height (h). The method of analysis is described and its limitations are discussed. The earth's magnetic field has been neglected in the calculations; although this may lead to an error in some of the actual heights quoted, it should not cause appreciable error in the variation of those heights.

Curves for the ‘international magnetically quietest days’ in each of the months analyzed are considered, and it is shown how, by a process of averaging, it is possible to deduce the form of a ‘mean quiet F layer’. This represents the basic behaviour of the region on magnetically quiet days without those details which are peculiar to any one day.

The resulting ‘mean quiet F layers’ are described in a series of curves. The detailed results, in tabulated form, are available from the Cavendish Laboratory for *bona fide* research workers.

1. INTRODUCTION

As the result of many years' work at a number of observing stations scattered widely over the earth, a large amount of experimental information is available concerning the F region of the ionosphere. This information is obtained in the first instance in the form of $h'(f)$ records, selected features from which are usually presented in tables of the critical frequencies of the F_1 and F_2 layers (f_0F2, f_0F1), the minimum virtual heights of the layers $h'F2$, $h'F1$, and sometimes the real height (h_m) of maximum electron density and the semi-thickness (y_m), deduced on the basis of some simple (usually parabolic) model.

The original $h'(f)$ records contain in addition sufficient information to specify the distribution of electron density (N) with height (h), subject only to some uncertainty arising from a lack of knowledge of the ionization between the layers. The problem of obtaining the $N(h)$ distribution from the experimental $h'(f)$ records has hitherto presented some difficulty and has involved considerable computation, particularly when the method used made no assumptions about the shape of the layer.

Recently, however, Kelso (1952) has suggested a numerical method of analysis which has proved sufficiently simple and rapid for application to an extensive series of records from a number of observatories. The resulting $N(h)$ curves contain information which is more useful in testing theories of the ionosphere than that available in the tables previously mentioned.

It is the purpose of this paper to describe how Kelso's computational method has been used to examine the electron distribution in the F region on magnetically quiet days at a series of places and times given in table 1 and to give a preliminary account of some of the results obtained. The detailed results of the calculations are available in tabulated form and may be obtained, by *bona fide* workers on the ionosphere, from the Radio Section, The Cavendish Laboratory, Cambridge, England.

TABLE 1. DETAILS OF THE PLACES AND MONTHS ANALYZED. THE MONTHLY AVERAGE RELATIVE ZURICH SUNSPOT NUMBER $\bar{R}$ IS INCLUDED FOR EACH MONTH

	sunspot maximum			sunspot minimum		
place	winter (northern)	equinox	summer (northern)	winter (northern)	equinox	summer (northern)
Slough						
geographic latitude 51·5° N geographic longitude 0·5° W geomagnetic latitude 54° N magnetic latitude 51·1° N inclination +68°	January 1950	March 1950	July 1950	December 1953	September 1953	June 1953
sunspot number	100	100	90	0	10	20
Watheroo						
geographic latitude 30·3° S geographic longitude 115·9° E geomagnetic latitide 41·8° S magnetic latitude 45·7° S inclination −64°	December 1939	March 1939	June 1940	December 1944	March 1944	June 1944
sunspot number	40	60	80	20	10	0
Huancayo						
geographic latitude 12° S geographic longitude 75·3° W geomagnetic latitude 0·6° S magnetic latitude 0° inclination 0°	December 1938	March 1939	June 1940	December 1944	March 1944	June 1944
sunspot number	90	60	80	20	10	0

We shall first, in §§2 and 3, give an account of the method of computation, and discuss its limitations and its accuracy. Next, in §4, we shall show how, by considering the five or ten magnetically quiet days in any one month, it is possible to describe a 'mean quiet F layer' typical of that month, and not unduly influenced by phenomena peculiar to any one of the days. Finally, in §5, we shall give a first-order description of the 'mean quiet F layer' for the places and times we have investigated. This description will take the form of curves showing the thickness and height of the layer, and the results will be compared with the values of $h'F$ published by the observatories concerned.

In collaboration with our colleagues we are making a more detailed examination of the results in the light of ionosphere theory and some of our conclusions are presented in the following paper (Ratcliffe, Schmerling, Setty & Thomas 1956).

2. The method of computation

The method used is based on that developed by Appleton (1930), de Groot (1930), Manning (1947, 1949), and Kelso (1952). It may be described, in outline, as follows. If the effects of the earth's magnetic field and of the collision of electrons with heavy particles are neglected, standard theory shows that

$$h(f_N) = \frac{2}{\pi}\int_0^{f_N} \frac{h'(f)\,\mathrm{d}f}{\sqrt{(f_N^2-f^2)}}, \tag{1}$$

where f = wave frequency,

$f_N = \sqrt{\left(\frac{Ne^2}{\pi\epsilon_0 m}\right)}$ represents the number density N of electrons in terms of the corresponding plasma frequency f_N,

$h'(f)$ = observed equivalent height at wave frequency f,

$h(f_N)$ = actual height at which the plasma frequency is f_N and the electron density is N.

Equation (1) is then written

$$h(f_N) = \frac{2}{\pi}\int_0^{\frac{1}{2}\pi} h'(f_N \sin\theta)\,\mathrm{d}\theta \tag{2}$$

by substituting $\theta = \sin^{-1}(f/f_N)$. The integration is now performed numerically by dividing the curve $h'(f_N\sin\theta)$ into a series of steps of equal width in θ. If five steps are taken, with equal increments in θ, they then correspond to values of f/f_N given by

$$f/f_N = 0{\cdot}156,\ 0{\cdot}454,\ 0{\cdot}707,\ 0{\cdot}891,\ 0{\cdot}988. \tag{3}$$

Equation (2) then becomes

$$h(f_N) = \frac{2}{\pi}\left\{\frac{1}{5}\,\frac{\pi}{2}\right\}\{h'(0{\cdot}156f_N)+h'(0{\cdot}454f_N)+h'(0{\cdot}707f_N)+h'(0{\cdot}891f_N)+h'(0{\cdot}988f_N)\}.$$

Ten steps correspond to the series

$$f/f_N = 0{\cdot}0785,\ 0{\cdot}2334,\ 0{\cdot}3827,\ 0{\cdot}5225,\ 0{\cdot}6494,\ 0{\cdot}7604,\ 0{\cdot}8526,\ 0{\cdot}9239,\ 0{\cdot}9724,\ 0{\cdot}9969. \tag{4}$$

In order, then, to calculate the height $h(f_N)$, at which f_N is the plasma frequency, it is merely necessary to read off from the experimental curve the values of $h'(f)$ at the frequencies given by the series (3) or (4), to add them up and to divide by the number of steps.

To facilitate reduction, the $h'(f)$ trace for the ordinary ray was replotted on a logarithmic frequency scale. By marking a card with the frequency ratios (3) or (4) on the same scale, the five or ten heights for each value of f_N could be immediately read off.

In practice it was found that the series (3) with five steps was sufficient for smooth single layers such as are observed at Slough at night throughout the year and on winter days. When the F region was bifurcated, the series (4) with ten steps was used.

3. Accuracy and limitations of the method

The lower frequencies in the series (3) or (4) often corresponded to echoes reflected from, or retarded in, the E layer, or to echoes which were removed by absorption. Under these circumstances the $h'(f)$ curve was copied from the record down to the minimum value of

h' in the F layer and was extrapolated to lower frequencies at the same value of h'. This leads to an overestimate in the height at which a given N is found, the error in the computed $N(h)$ curve being negligible for the greater values of h, though there might be an appreciable error for the smaller values. In particular, the shapes deduced for the F_1 layer might not be correct. This paper, however, deals only with the F_2 layer.

The omission of the effect of the earth's magnetic field results in a systematic error in the computed electron distribution and leads to heights which are too great. When our work started no simple method was available for including this effect, and we have therefore made all the calculations as though no field were present.

We have since had the advantage of seeing in advance some work of Dr D. H. Shinn's, in which the effect of the earth's field is included. In this method the coefficients of the series (3) or (4) are replaced by modified coefficients which depend on the magnetic field at the place concerned. Some of the results have been recalculated by this method for comparison with results obtained neglecting the field. The $N(h)$ curves calculated with and without the earth's magnetic field coincide at their lower edges, but for higher frequencies the distribution calculated without the field lies above that calculated with the field present. The thickness of the layer calculated without the field varies from about 1·3 (for low values of the critical frequency) to about 1·1 (for high values of the critical frequency) times the thickness with the field.

A first-order correction for the effect of the earth's field may be made by reducing the heights calculated at Slough and Watheroo by about 10 km. At Huancayo the calculated distribution obtained is independent of the earth's field.

Although the absolute values of height and thickness of the F_2 layer given in this paper are subject to the limitations discussed above, it seems unlikely that conclusions based on the changes of height and thickness (with which we are often concerned in ionospheric theory), will be seriously in error.

4. THE 'MEAN QUIET F LAYER'

In the work here described we have restricted our attention to the F layer at times of small magnetic disturbance, in the hope that, in this way, we might gain information about its behaviour under fundamentally simple conditions. In any one month we therefore analyzed the $h'(f)$ records obtained on the five (or ten) international magnetically quiet days.* It was then found that the results, even for these quiet days, showed random variations which were not the same from day to day. As an example of these we show in figure 1 the electron distributions $N(h)$ calculated for each hour at Slough on 12 January 1950. The same information is plotted differently in figure 2 in the form of $N(t)$ curves which show at a series of different heights how N varied with t. The dashed line in figure 1 shows the 250 km $N(t)$ curve included in figure 2. It is noticeable that, on the $N(t)$ curves, oscillations are superimposed on the smooth daily variation. Comparison with the $N(h)$ curves shows that these oscillations are the result of contraction and expansion of the layer, and are not caused by variations in the strength of the ionizing radiation. This fact has been noticed previously (Ratcliffe 1951). The important point here is that oscillations of this kind occur even on

* These are listed regularly in the *Journal of Geophysical Research* (see, for example, vol. **58**, p. 110 (1953)).

magnetically quiet days, and that there does not seem to be any obvious relation between the oscillations on different days. It appears that they represent a random type of disturbance which is always present.

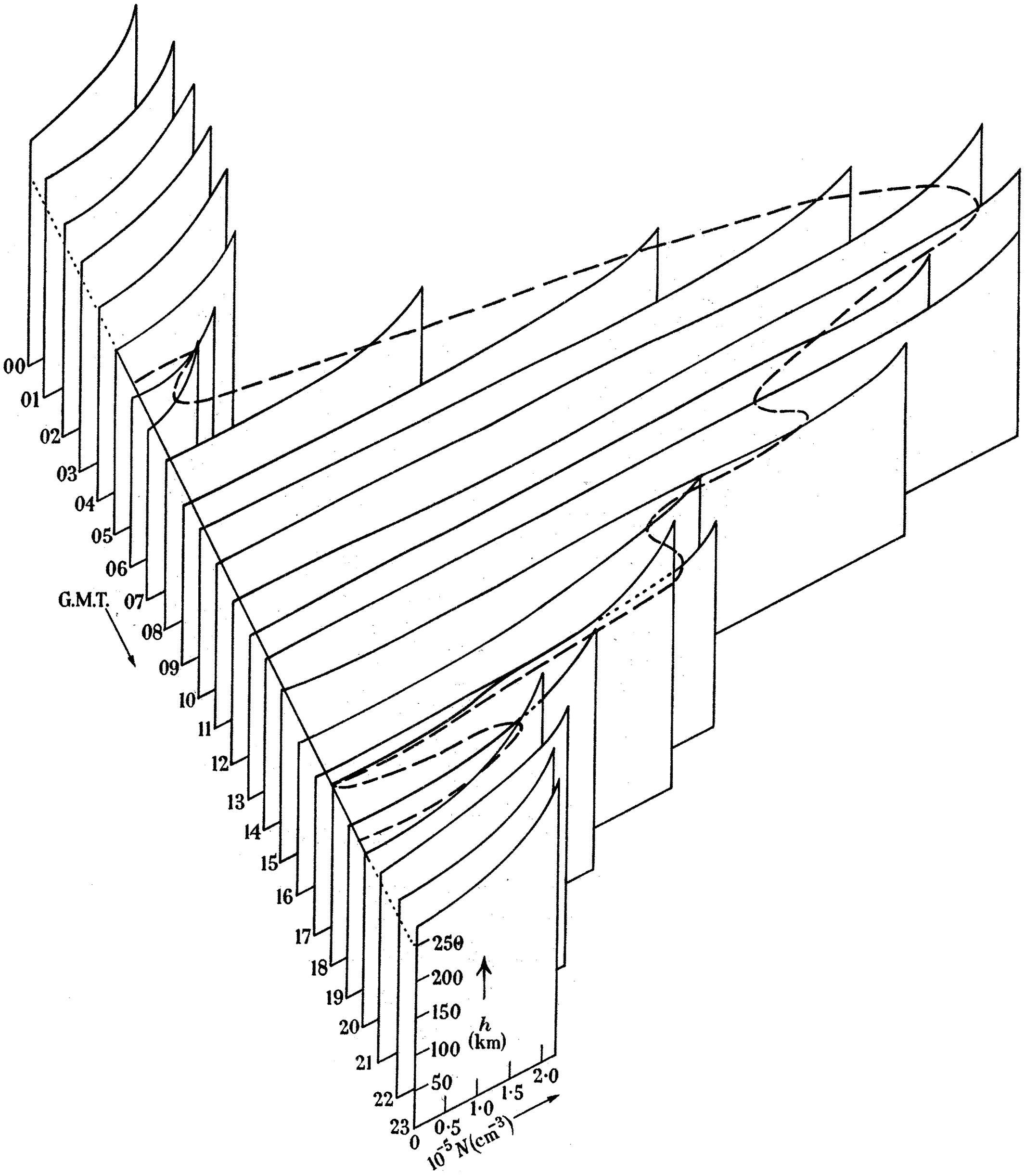

FIGURE 1. $N(h)$ distribution for every hour at Slough on 12 January 1950. The dashed line gives the variation of N with time at 250 km ($N(t)$ curve).

For the purpose of fundamental theory it is desirable to know how the quiet ionosphere would behave if these random variations were not present. We have therefore attempted to deduce, from a series of magnetically quiet days, a 'mean quiet F layer' in which the effect of the random variations has been reduced by a process of averaging.

Figure 3 shows the average $N(t)$ curves for nine of the international magnetically quiet days at Slough in January 1950. The single day represented by the curves of figure 2 is included among these nine, and it is apparent how the averaging process has reduced the amount of the random variation. In order to test the usefulness of averaging in this way, and in order to find how many quiet days are required to give a reasonable average, 10 days for September 1950 at Slough were split into two sets of 5, and average $N(t)$ curves calculated for each set separately are shown in figure 4. It is clear that an average of 5 days is sufficient to remove most of the effect of the random variations, and that the smooth curve for one set of 5 quiet days is quite closely the same as that for another set of 5.

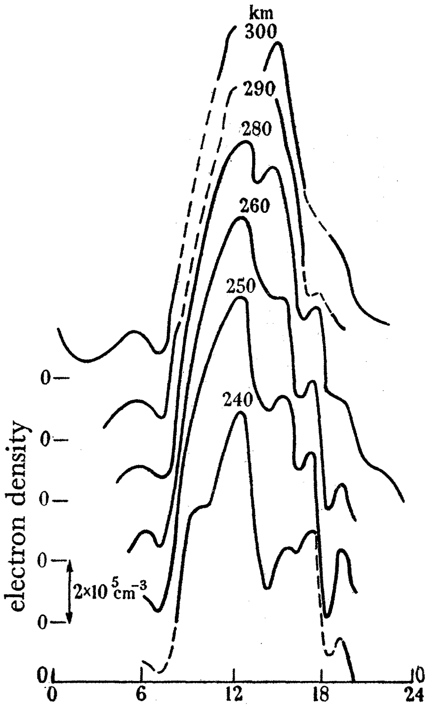

FIGURE 2

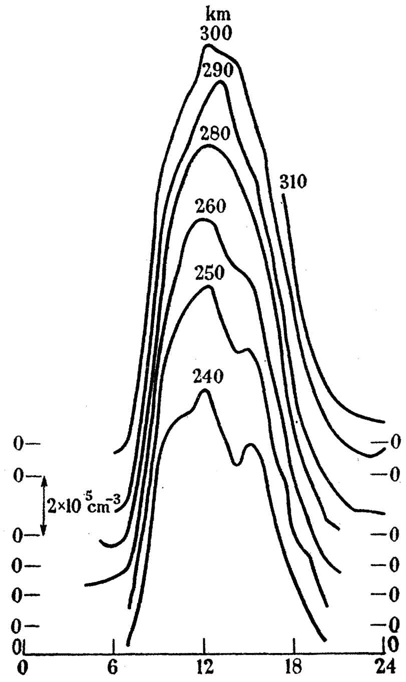

FIGURE 3

FIGURE 2. Series of $N(t)$ curves showing how N changed with time at different heights on 12 January 1950 at Slough. The curve labelled 250 km corresponds to the dashed line in figure 1.

FIGURE 3. Average $N(t)$ curves for 9 international quiet days at Slough in January 1950.

The distance indicated by the arrows on the ordinate scales corresponds to $N = 2\times10^5\,\mathrm{cm}^{-3}$. The separate curves have different zeros as indicated by the zeros at the sides.

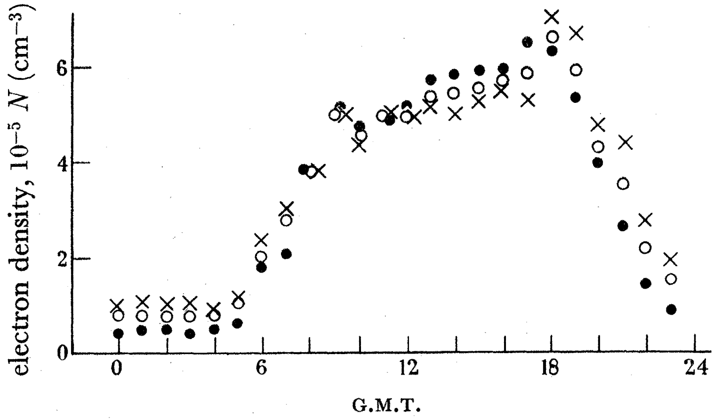

FIGURE 4. $N(t)$ curves for 300 km at Slough in September 1950. ○ Mean of 10 international quiet days. ● Mean of 5 international quiet days. × Mean of 5 international quiet days.

When the average $N(t)$ curves obtained in this way are considered in more detail they are, however, found to be unsatisfactory for the following reason. If $N(h)$ curves are constructed from the average $N(t)$ curves they are often found to have the form shown in figure 5 (which is derived from figure 3). Some of these curves are not smooth, although all the $N(h)$ curves representing the original data were smooth. It is clear that, in the averaging, too much attention has been paid to the $N(t)$ curves and not enough to the $N(h)$ curves. A better method of averaging was therefore sought, which would provide average $N(t)$ and $N(h)$ curves in which equal attention had been paid to the two types of curve.

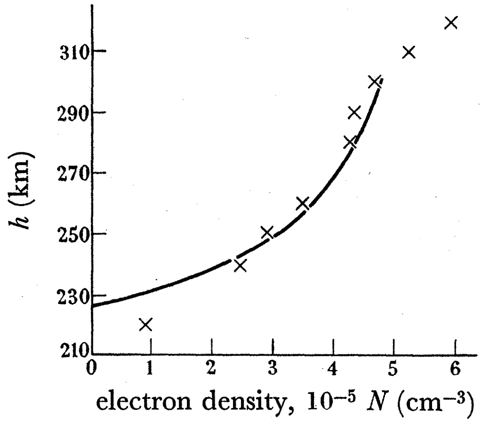

Figure 5. To illustrate the two methods of averaging. The crosses are taken from averaged $N(t)$ curves for January 1950 (figure 3) at 1800 G.M.T. The line represents the 'mean quiet F layer' calculated as described in the text. (Slough, January 1950 at 1800 G.M.T.)

The method finally chosen is illustrated in figure 6. For one given hour the $N(h)$ curves appropriate to a series of days ((*a*) in figure 6) were normalized so that N was expressed as a fraction of its magnitude (N_m) at the peak of the layer. The resulting curves are shown at (*b*). The normalized curves were then averaged by finding the average heights at which a series of values of N/N_m were found. The resulting curve is shown at (*c*) with the upper scale of abscissae. Next, the average value ($\overline{N}_m$) of the electron densities (N_m) at the peaks of the layers in (*a*) was calculated, and the scale of the average curve (*c*) was re-labelled so that its peak fell at $\overline{N}_m$. This new scale is the lower one in (*c*). The process was repeated for each hour.

When separate F_1 and F_2 layers were observed, their maximum electron densities N_mF1 and N_mF2 varied differently. The two layers were therefore normalized and averaged separately, and were then plotted together as a composite curve. Sometimes it was necessary to make small alterations in the lower part of the F_2 curve to make it fit the F_1 curve.

The results discussed in §5 all refer to 'mean quiet F layers' computed in this way. The curves obtained were found to be smoother than the curves for individual days, and curves deduced from two different sets of 5 quiet days agreed more closely with each other than with the curves for individual days. It is therefore considered that the averages obtained in this way have some real meaning for theories of the F region, and they will be said to describe a 'mean quiet F layer'.

The work involved in computing the 'mean quiet F layer' can be considerably reduced by performing the averaging process on the original $h'(f)$ curves instead of on the $N(h)$

curves deduced from them. The procedure is then similar to that illustrated in figure 6, except that we are concerned with h' and f as the axes instead of h and N. One considerable simplification then arises because the curves have already been plotted with a logarithmic scale for f, so that the change corresponding to the transition from (*a*) to (*b*) in figure 6 is now performed simply by using a scale marked with fractional values of the critical frequency on the same logarithmic scale. Average $N(h)$ curves constructed in this way direct from average $h'(f)$ curves were compared with those constructed by first finding the individual $N(h)$ curves and then averaging them, and no significant differences were found.

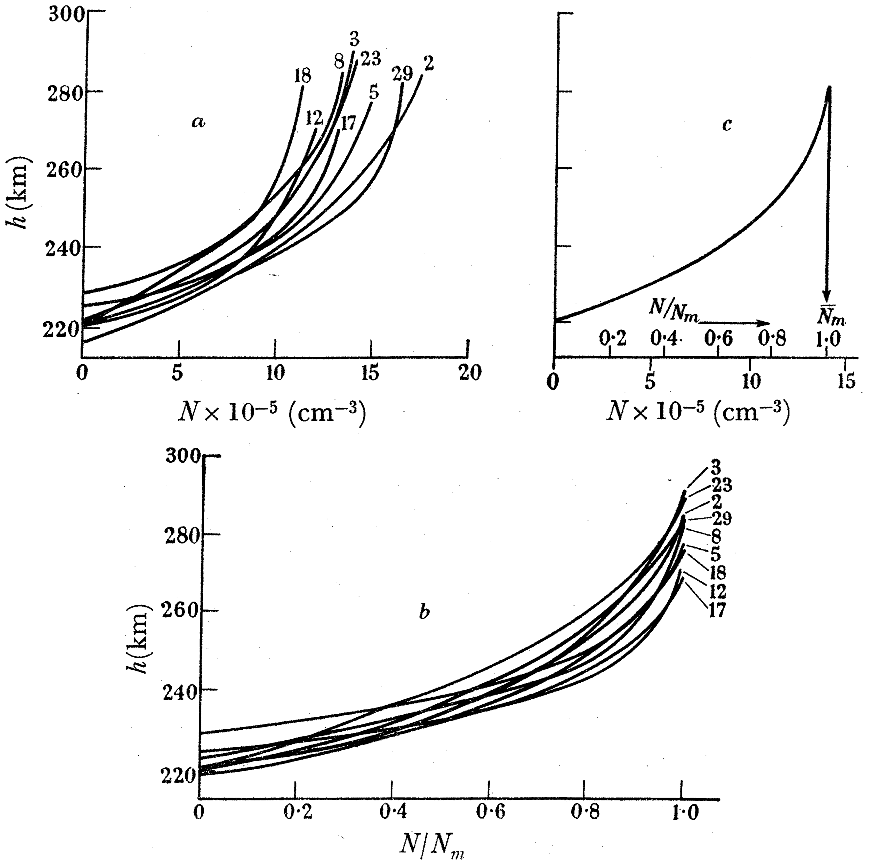

FIGURE 6. To illustrate the method of averaging. (*a*) Noon $N(h)$ curves at Slough on 9 days in January 1950. (*b*) The same curves normalized along the axis of N. (*c*) The curves of (*b*) averaged in height. The upper scale is the normalized one, and the lower is arranged to bring the peak of the layer to $\bar{N}_m$.

5. RESULTS AND DISCUSSION

In an attempt to survey the form of the F layer over the world as a whole, at a series of representative times, the records made at the places and in the months listed in table 1 were analyzed. The months were chosen to represent summer, equinox and winter, near sunspot maximum and sunspot minimum. We are much indebted to Dr Smith-Rose, Director of Radio Research of the Department of Scientific and Industrial Research, London, for the loan of the records from Slough, and to Dr M. A. Tuve, Director of the Department of Terrestrial Magnetism, Washington, D.C., U.S.A., for providing tracings of the records from Huancayo and Watheroo.

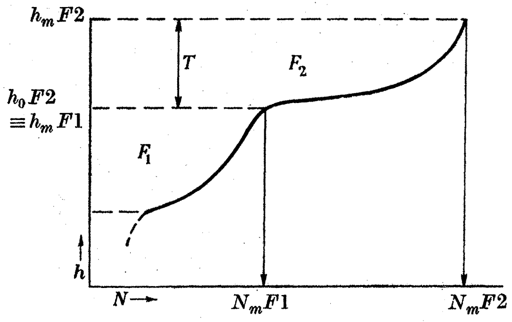

Figure 7. Schematic $N(h)$ curve showing parameters used to describe the layer characteristics.

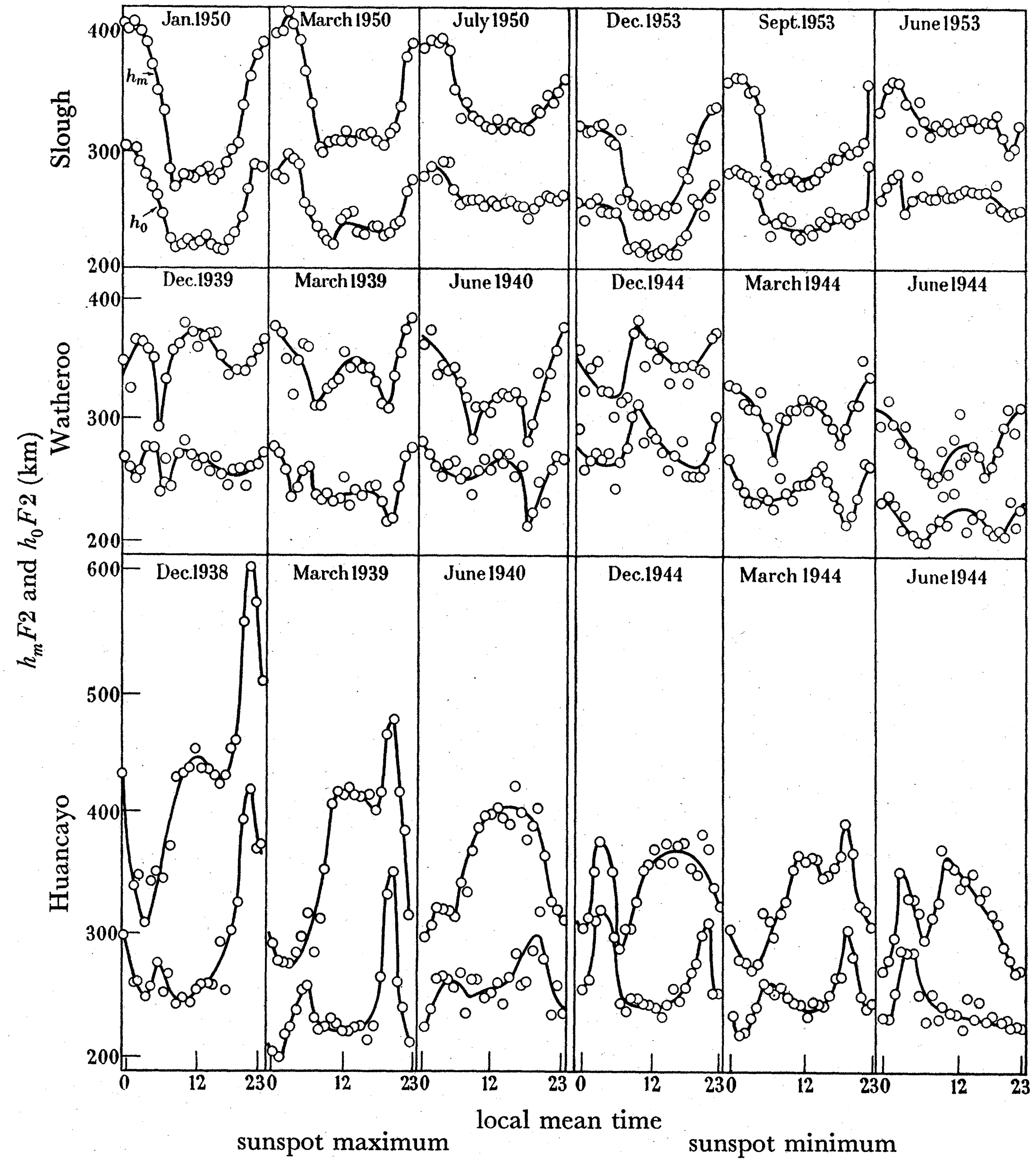

Figure 8. Diurnal variation of h_mF2 and h_0F2 at Slough, Watheroo and Huancayo at different seasons and at two epochs of the sunspot cycle.

The days chosen for analysis were those listed as the five magnetically quietest days in the appropriate months. In some months the ten magnetically quietest days were analyzed.

For each month, and place, the 'mean quiet F layer' was computed in the way described in §4. The tabulated results are available for other workers as described in §1. Here we shall present a few curves which summarize some of the main points.

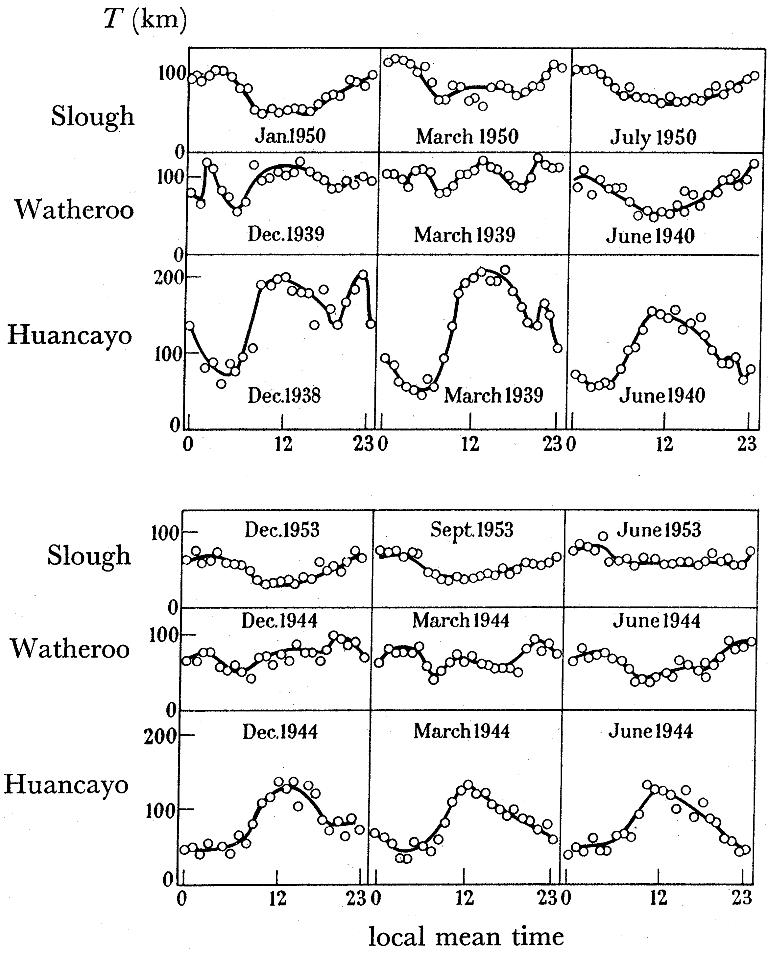

FIGURE 9. Diurnal variation of F_2 layer thickness (T) at Slough, Watheroo and Huancayo at different seasons and at two epochs of the sunspot cycle, namely (top) maximum and (bottom) minimum.

We shall describe the 'mean quiet F layer' in terms of the quantities shown in figure 7. It should be noted particularly that the symbol h_0F2 is used for the true height of the bottom of the F_2 layer. When the F_1 layer is present this height corresponds to the level at which the F_1 and F_2 layers merge. The thickness of the layer (h_mF2-h_0F2) is denoted by T. Figures 8 and 9 show how these quantities vary with time of day at the different places and seasons.

Many of the smaller variations of the points plotted in the figures seem to be significant, because they are repeated in the separate days which go to make up the 'mean quiet F layer', but in drawing the curves through the points we have only emphasized the major variations.

Figure 10 shows the relation between our results and the minimum virtual height ($h'F2$) of the F_2 layer, which is published by several ionospheric observatories. The lines show the diurnal variation of the published monthly average values of $h'F2$, and the circles show our values of h_0F2 (for the international quiet days analyzed) as indicated on figure 7.

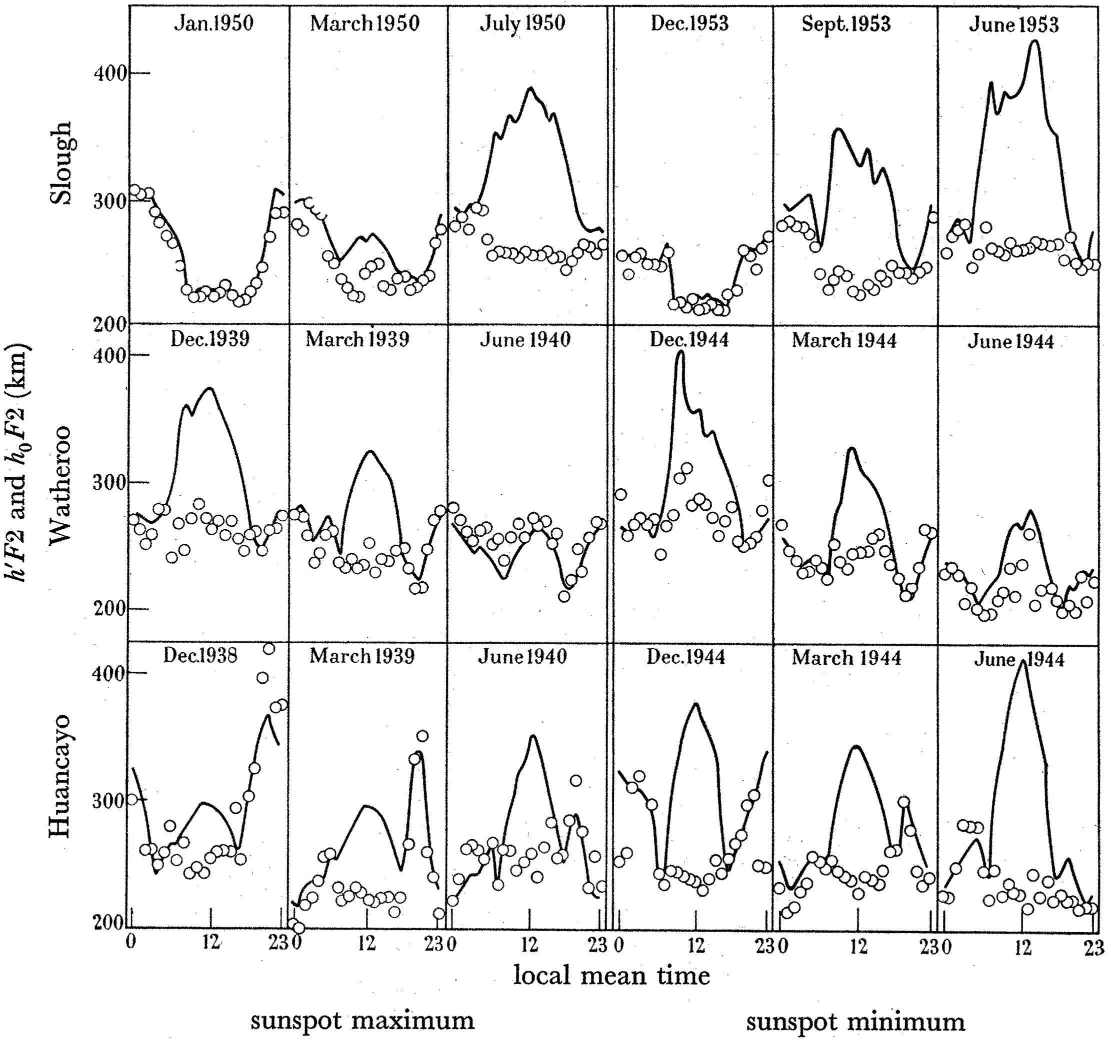

FIGURE 10. Comparison of $h'F2$ and h_0F2 (see figure 7). The lines show the mean monthly diurnal variation of $h'F2$ at Slough, Watheroo and Huancayo at different seasons and at two epochs of the sunspot cycle. The circles represent values of h_0F2 taken from figure 8.

When the F_1 layer is present the value of $h'F2$ seems to be misleading. Examination of the $h'(f)$ records for these months emphasizes that, as most workers realize, the quantity $h'F2$ is too much influenced by group retardation in the F_1 layer for it to be of much use as an indication of the height of the F_2 layer.

This work was carried out as part of a programme of research aided by a grant from the Department of Scientific and Industrial Research.

One of us (E.R.S) wishes to acknowledge gratefully the receipt of a Department of Scientific and Industrial Research Maintenance Award.

The authors wish to thank the computing staff of the Radio Group of the Cavendish Laboratory, and, in particular, Mrs J. W. King, who were responsible for the large amount of computational work involved. They are also indebted to Dr D. H. Shinn for helpful discussion.

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